THINK AND DISCUSS

1. Show two ways to evaluate \( \sqrt{16(9)} \).

2. Show two ways to evaluate \( \sqrt{\frac{100}{4}} \).

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write the property and give an example.

<table>
<thead>
<tr>
<th>Product Property of Square Roots</th>
<th>Quotient Property of Square Roots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Words</td>
<td>Example</td>
</tr>
</tbody>
</table>

11-6 Exercises

GUIDED PRACTICE

1. Vocabulary In the expression \( \sqrt{3x - 6} + 7 \), what is the radicand?

Simplify each expression.

2. \( \sqrt{81} \)  
3. \( \sqrt{\frac{98}{2}} \)  
4. \( \sqrt{(a + 7)^2} \)

Simplify. All variables represent nonnegative numbers.

5. \( \sqrt{180} \)  
6. \( \sqrt{40} \)  
7. \( \sqrt{648} \)  
8. \( \sqrt{m^2n^3} \)  
9. \( \sqrt{32x^4y^3} \)  
10. \( \sqrt{200a^2b} \)  
11. \( \sqrt{\frac{17}{25}} \)  
12. \( \sqrt{\frac{7}{16}} \)  
13. \( \sqrt{\frac{6}{49}} \)  
14. \( \sqrt{\frac{b}{c^2}} \)  
15. \( \sqrt{\frac{4x^2}{36x}} \)  
16. \( \sqrt{\frac{7a^4}{9a^3}} \)  
17. \( \sqrt{\frac{108}{49}} \)  
18. \( \sqrt{\frac{204}{25}} \)  
19. \( \sqrt{\frac{512}{81}} \)  
20. \( \sqrt{\frac{1}{36x^2}} \)  

21. \( \sqrt{\frac{50x^2}{169}} \)  
22. \( \sqrt{\frac{72x^2}{4x^4}} \)

23. Recreation Your boat is traveling due north from a dock. Your friend's boat left at the same time from the same dock and is headed due east. After an hour, your friend calls and tells you that he has just stopped because of engine trouble. How far must you travel to meet your friend? Give your answer as a radical expression in simplest form. Then estimate the distance to the nearest mile.
PRACTICE AND PROBLEM SOLVING

24. \( \sqrt{100} \)  
25. \( \sqrt{\frac{800}{2}} \)  
26. \( \sqrt{3^2 + 4^2} \)  
27. \( \sqrt{3 \cdot 27} \)
28. \( \sqrt{a^4} \)  
29. \( \sqrt{(x + 1)^2} \)  
30. \( \sqrt{(5 - x)^2} \)  
31. \( \sqrt{(x - 3)^2} \)

Simplify. All variables represent nonnegative numbers.

32. \( \sqrt{125} \)  
33. \( \sqrt{4000} \)  
34. \( \sqrt{216a^2b^2} \)  
35. \( \sqrt{320r^2s^2} \)
36. \( \sqrt{\frac{15}{64}} \)  
37. \( \sqrt{\frac{45}{4}} \)  
38. \( \sqrt{\frac{64a^4}{4a^6}} \)  
39. \( \sqrt{\frac{14z^5}{9z^3}} \)
40. \( \sqrt{\frac{128}{81}} \)  
41. \( \sqrt{\frac{x^3}{y^6}} \)  
42. \( \sqrt{\frac{150}{196x^2}} \)  
43. \( \sqrt{\frac{192x^3}{49s}} \)

44. Amusement Parks A thrill ride at an amusement park carries riders 160 feet straight up and then releases them for a free fall. The time \( t \) in seconds that it takes an object in free fall to reach the ground is \( t = \sqrt{\frac{d}{16}} \), where \( d \) is the distance in feet that it falls. How long does it take the riders to reach the ground? Give your answer as a radical expression in simplest form. Then estimate the answer to the nearest tenth of a second.

45. \(-4\sqrt{75}\)  
46. \(-\sqrt{80}\)  
47. \(5x\sqrt{63}\)  
48. \(3\sqrt{48x}\)

49. \(2\sqrt{\frac{x^2}{4}}\)  
50. \(\frac{1}{2}\sqrt{\frac{1}{25}}\)  
51. \(3x\sqrt{\frac{x^5}{81}}\)  
52. \(\frac{12}{x}\sqrt{\frac{x^2y}{36}}\)

Use the Product Property or the Quotient Property of Square Roots to write each expression as a single square root. Then simplify if possible.

53. \(\sqrt{12\sqrt{3}}\)  
54. \(\sqrt{18\sqrt{8}}\)  
55. \(\sqrt{10\sqrt{5}}\)  
56. \(\sqrt{8\sqrt{14}}\)

57. \(\sqrt{\frac{33}{11}}\)  
58. \(\sqrt{\frac{24}{2}}\)  
59. \(\sqrt{\frac{60}{\sqrt{3}}}\)  
60. \(\sqrt{\frac{72}{9}}\)

61. Multi-Step How many whole feet of fencing would be needed to enclose the triangular garden that is sketched at right? Explain your answer.

62. Write About It Write a series of steps that you could use to simplify \(\sqrt{\frac{28}{49}}\).

63. This problem will prepare you for the Multi-Step Test Prep on page 830.
   a. The vertical component of a roller coaster's speed in feet per second at the bottom of a hill is \( v = \sqrt{64h} \), where \( h \) is the hill's height in feet. Simplify this expression. Then estimate the velocity at the bottom of a 137-foot hill.
   b. The distance along the track of a hill is \( d = \sqrt{x^2 + h^2} \), where \( x \) is the horizontal distance along the ground and \( h \) is the hill's height. Where does this equation come from?
   c. For the hill in part a, the horizontal distance along the ground is 103 feet. What is the distance along the track? Round your answer to the nearest tenth.
Product Property of Radical Expressions \( \rightarrow \sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \) (and)

\[ \sqrt{ab} \leftarrow \text{radicand} \]

4, 9, 16, 25 ...

**Hint:** Think in terms of perfect squares factors whenever possible to simplify radical expressions

**Hint:** Think about divisibility rules.

**EX:** Sum of the digits for divisibility by 9 and 3

**EX:** If last 2 digits divisible by 4, the entire number is divisible by 4

***Assume all variables represent nonnegative numbers.***

### A) General Rules can be seen in these 6 problems:

1. \( \sqrt{5} \cdot \sqrt{5} \)
   
   \[
   \begin{align*}
   \sqrt{25} &= 5 \\
   \end{align*}
   \]

2. \( \sqrt{8} \cdot \sqrt{8} \)

3. \( \sqrt{a} \cdot \sqrt{a} \)

4. \( \sqrt{4^2} \)

5. \( \sqrt{x^2} \)

6. \( \sqrt{b^2} \)

### B) Product Property of Square-Roots \( \sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \)

***Assume all variables represent nonnegative numbers***

1. \( \sqrt{18} \)

2. \( 3\sqrt{25} \)
3. \( \sqrt{180} \)
\[
\sqrt{9} \cdot \sqrt{20} \\
3 \cdot \sqrt{20} \\
3 \cdot \sqrt{4} \cdot \sqrt{5} \\
6 \sqrt{5}
\]

4. \( \sqrt{x^3y^2} \)
\[
\sqrt{x^2 \cdot x \cdot y^2} \\
x \cdot \sqrt{x \cdot y} \\
x y \sqrt{x}
\]

5. \( \sqrt{48a^4b^5} \)
\[
\sqrt{16 \cdot a^4 \cdot b^5} \\
\sqrt{16} \cdot \sqrt{a^4} \cdot \sqrt{b^5} \\
4 \cdot (\sqrt{a} \cdot \sqrt{a} \cdot \sqrt{a} \cdot \sqrt{a}) \cdot \sqrt{b} \cdot \sqrt{b} \cdot \sqrt{b} \\
4 \sqrt{3} \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \\
4a^2b^2 \sqrt{3} \cdot \sqrt{b} \\
4a^2b^2 \sqrt{3b}
\]

6. \( \sqrt{96a^2b^2} \)
\[
\sqrt{16 \cdot 6 \cdot a^2 \cdot b^2} \\
2 \cdot \sqrt{16} \cdot a \cdot b \\
2a \sqrt{6} \\
2a \sqrt{4 \cdot \sqrt{6}} \\
2a \cdot 2 \sqrt{6} \\
2 \cdot 2a \sqrt{6} \\
4a \sqrt{6}
\]

7/8. Pythagorean Theorem: \( a^2 + b^2 = c^2 \)

Find the missing side as a simplified radical and to the nearest tenth.

8

\( \text{hypotenuse} \)

\[
\begin{align*}
8^2 &+ 12^2 = c^2 \\
144 &+ 144 = c^2 \\
\sqrt{288} &\approx c \\
14.4 &\text{ units} = c
\end{align*}
\]

12

\[
\begin{align*}
2\sqrt{52} \text{ units} &\approx c \\
2 \cdot 2\sqrt{13} &\approx c \\
14.4 &\text{ units} = c
\end{align*}
\]

14

\[
\begin{align*}
a^2 + 144 &= 196 \\
a^2 &= 52 \\
\sqrt{52} &\approx a \\
7.07 &\text{ units} = a
\end{align*}
\]